Indian Statistical Institute, Bangalore

B. Math. First Year First Semester

Analysis -I

Back Paper Examination | Date: 01-01-2018

Maximum marks: 100 | Time: 3 hours

Instructor: B V Rajarama Bhat

(1) Show that every bounded sequence of real numbers has a convergent subsequence. [15]

(2) Obtain lim sup and lim inf for following real sequences:

(a) $\{a_n\}_{n\geq 1}$ where $a_n = \frac{6-2n+n^2}{8-2n^2}$;

(b) $\{b_n\}_{n\geq 1}$ where $b_n = 0$ for n odd and $b_n = 2 - \frac{100}{n^2}$ for n even;

(c) $\{c_n\}_{n\geq 1}$ where $c_n = a_n + b_n$ for $n \geq 1$, where a_n, b_n are as above.

[15]

(3) Suppose $\{c_n\}_{n\geq 1}$ is a sequence of strictly positive real numbers converging to a strictly positive real number c. Show that the sequences $\{\frac{1}{k}(c_1 + c_2 + \cdots + c_k)\}_{k\geq 1}$ and $\{(c_1 c_2 \cdots c_k)^{\frac{1}{k}}\}_{k\geq 1}$ converge to c. [15]

(4) Suppose $u : [0,1] \to \mathbb{R}$ is a continuous function. Define a new function $v : [0,1] \to \mathbb{R}$ by

$$v(x) = \begin{cases} x & \text{if } u(x) \geq x; \\ u(x) & \text{if } u(x) \leq x. \end{cases}$$

Show that v is continuous. [15]

(5) Suppose $f, g : [a,b] \to \mathbb{R}$ are differentiable at $c \in (a,b)$ and $g(x) \neq 0$ for every x. Show that $h := \frac{f}{g}$ is differentiable at c, and obtain a formula for $h'(c)$. [15]

(6) Let $f : [0,1] \to \mathbb{R}$ be a differentiable function. Suppose $f(0) = 0$ and $f'(x) \geq f(x)$ for all $x \in [0,1]$. Show that $f(x) \geq 0$ for all x. (Hint: Consider g defined by $g(x) = e^{-x} f(x)$. [15]

(7) State and prove Taylor's theorem. [15]